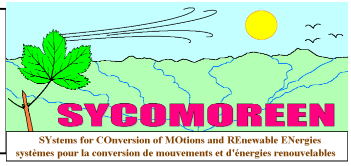
Kinetic limits of wind propulsion while facing the wind

Precisions on http://sycomoreen.free.fr

calculated notably for the *Pascal HA PHAM's* ***ROBIPLAN*** **turbine**

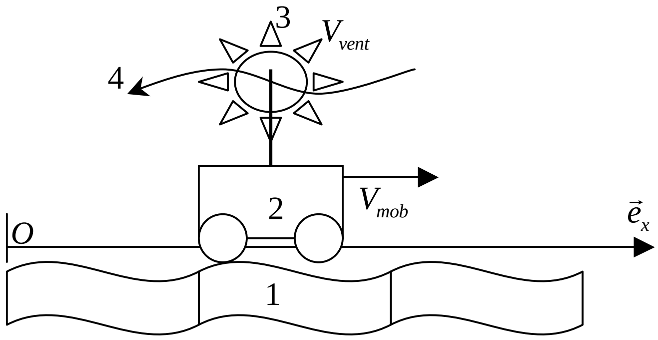


Setting of the context

One considers a mobile which moves facing the wind with the only power of the wind *via* a turbine.

There is *4 entities in relation* :

1. The ground which supports the motion.
2. The mobile which translates itself
3. The turbine mounted on the mobile
4. The wind crossing the turbine.



All these elements get speeds relatively to a fixed referential *Rg* which is supposed galilean. In this survey, the reference $(O,\vec{e}_x)$ is the only axis of motion between the 4 previous entities. One will suppose that the ground (1) is immobile and that the speeds are *positive by convention* relatively to *Rg* :

- ***Vmob*** absolute speed of the mobile (2)
- ***Vwind*** absolute speed of the wind (4)

Calculation of the limits

A mobile moves against the wind with the only power of it extracted from a wind-turbine. *The drag of the turbine is taken in account as the only strentgh which works against the motion* and one supposes too that the recovered power from the turbine is totally and perfectly transmitted to the mobile.

The kinetic incidental power on the turbine is : $P_{kin} = \frac{1}{2}\rho S\left(V_{mob}+V_{wind}\right)^3$

The mechanical recovered power is : $P_{mot} = \eta P_{kin} = \frac{\eta}{2}\rho S\left(V_{mob}+V_{wind}\right)^3$ where η is the wind-output

The dragging strentgh coming from the slowing of the wind is : $F_t = \rho S\left(V_{mob}+V_{wind}\right)^2(1-k)$

where k is the slowing factor, while the wind is crossing the turbine, its speed collapse **relatively** to the mobile.

The braking power due to the drag of the turbine is : $P_t = F_t V_{mob} = \rho S V_{mob}\left(V_{mob}+V_{wind}\right)^2(1-k)$

The top speed limit is attempt while the mechanical power is the same as the braking power :

$$\frac{\eta}{2}\rho S\left(V_{mob}+V_{wind}\right)^3 = \rho S V_{mob}\left(V_{mob}+V_{wind}\right)^2(1-k)$$

either $\frac{\eta}{2}\left(V_{wind}+V_{mob}\right) = V_{mob}(1-k) \Leftrightarrow \left(V_{wind}+V_{mob}\right) = V_{mob}\frac{2(1-k)}{\eta}$

thus $\frac{V_{wind}}{V_{mob}}+1 = \frac{2(1-k)}{\eta}$ and finally $\boxed{V_{mob} = \frac{1}{\frac{2(1-k)}{\eta}-1}V_{wind}}$

SYNTHESIS OF RESULTS

Wind-turbine	V_{mob}/V_{wind}
MONOROBI	25 %
TRI-BLADE	56 %
BIROBI	66 %
ULTIMATE	80 %

ROBIPLAN's limits : η is optimum when the wind is slowed 2 times (k = 1/2)

For MONOROBI : $\eta = \frac{2}{10}$ and $\boxed{V_{mob} = \frac{1}{4}V_{wind}}$

For BIROBI : $\eta = \frac{4}{10}$ and $\boxed{V_{mob} = \frac{2}{3}V_{wind}}$

Limit of tri-blade turbine : $\eta = \frac{48}{100}$ is optimum when the wind is slowed 3 times (k = 1/3) $\boxed{V_{mob} = \frac{9}{16}V_{wind}}$

Ultimate limit : $\eta = \frac{16}{27}$ maximum of Betz when the wind is slowed 3 times (k = 1/3) $\boxed{V_{mob} = \frac{4}{5}V_{wind}}$